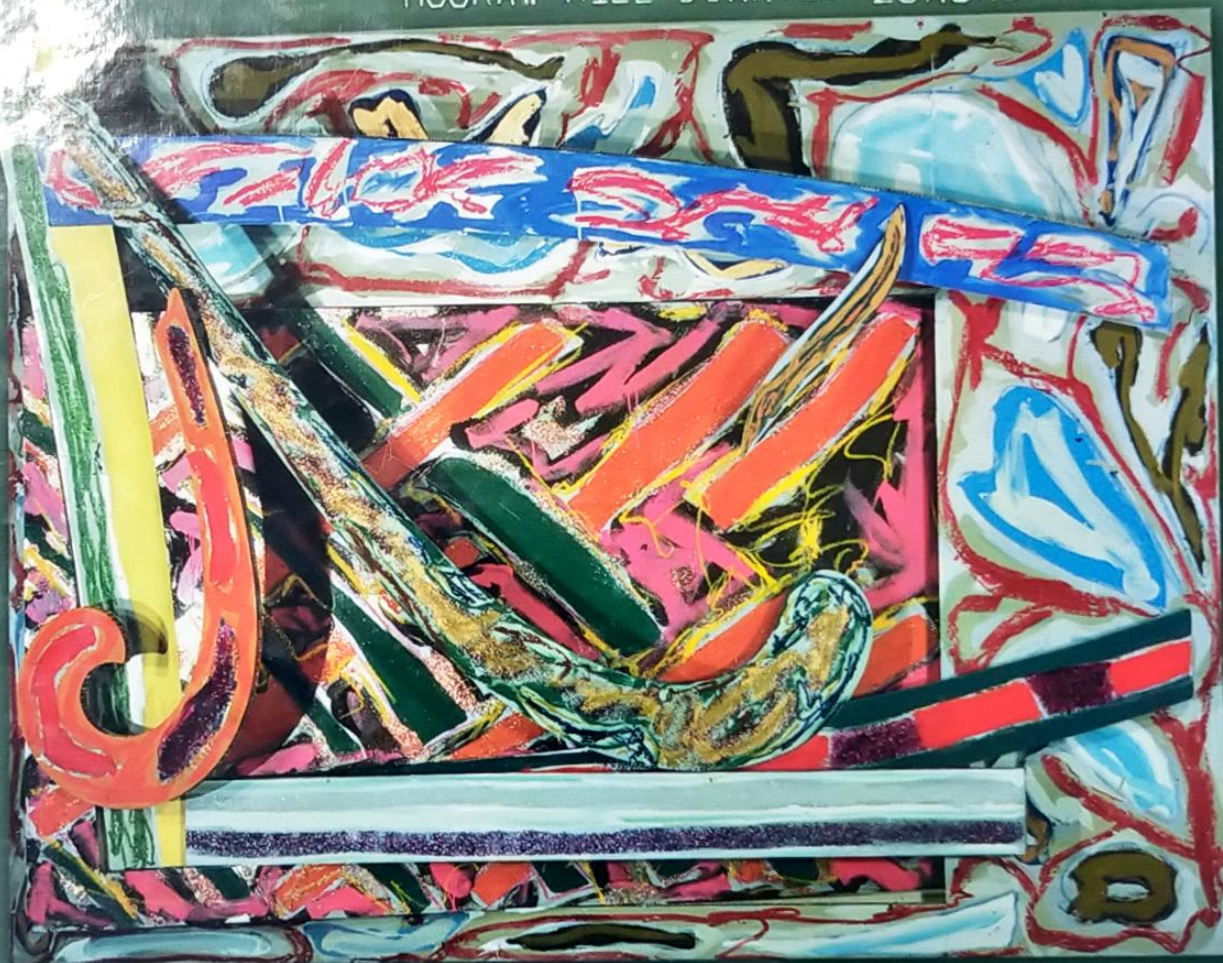


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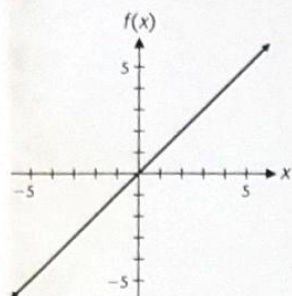
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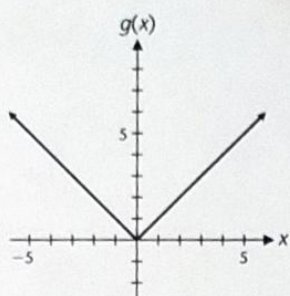
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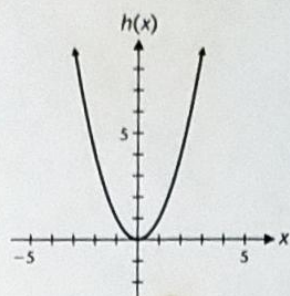
CHAPTER 1 BASIC FUNCTIONS



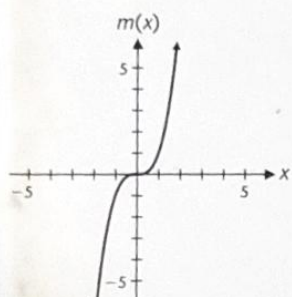
Identity function  
 $f(x) = x$



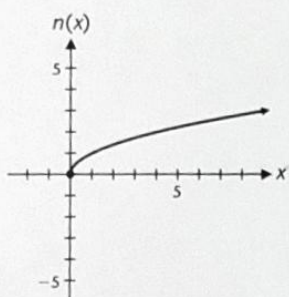
Absolute value function  
 $g(x) = |x|$



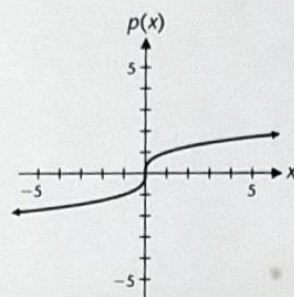
Square function  
 $h(x) = x^2$



Cube function  
 $m(x) = x^3$

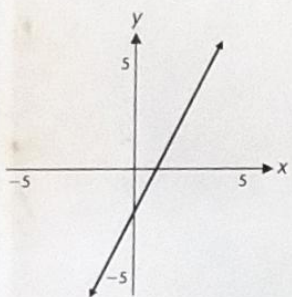


Square root function  
 $n(x) = \sqrt{x}$

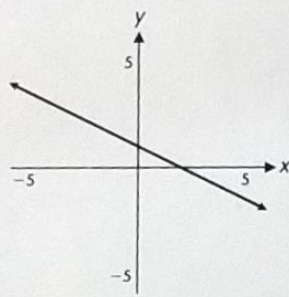


Cube root function  
 $p(x) = \sqrt[3]{x}$

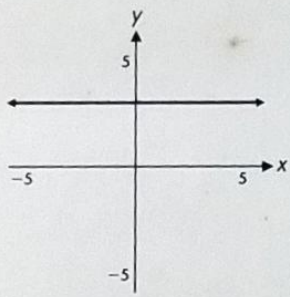
CHAPTER 2 LINEAR AND QUADRATIC FUNCTIONS



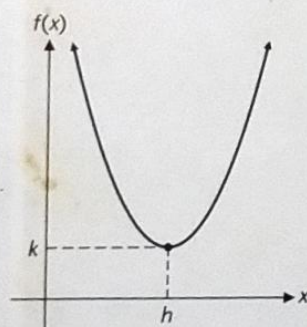
Linear function  
 $f(x) = 2x - 2$



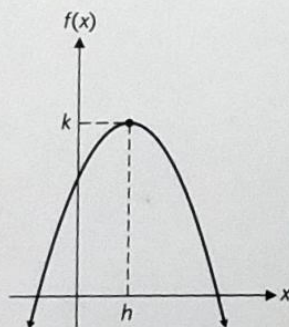
Linear function  
 $g(x) = -0.5x + 1$



Constant function  
 $h(x) = 3$



$a > 0$   
Opens upward



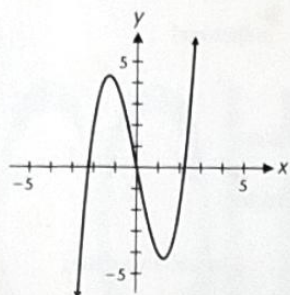
$a < 0$   
Opens downward

Quadratic function  
 $f(x) = ax^2 + bx + c = a(x - h)^2 + k_1 \quad a \neq 0$

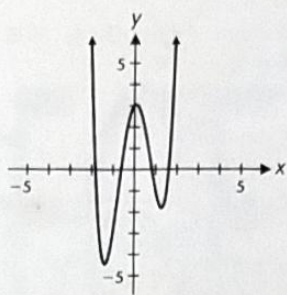
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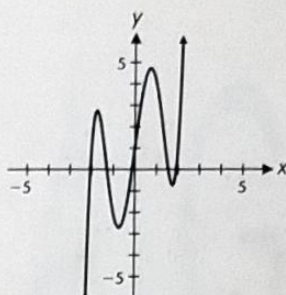
### CHAPTER 3 POLYNOMIAL AND RATIONAL FUNCTIONS



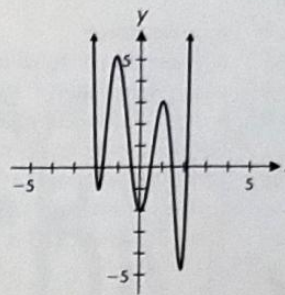
Third-degree polynomial  
 $g(x) = x^3 - 5x$



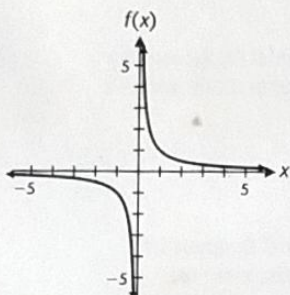
Fourth-degree polynomial  
 $G(x) = 2x^4 - 7x^2 + x + 3$



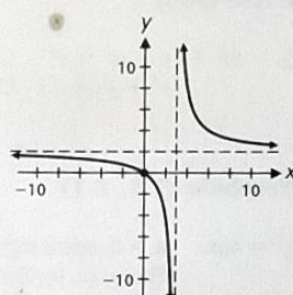
Fifth-degree polynomial  
 $h(x) = x^5 - 6x^3 + 8x + 1$



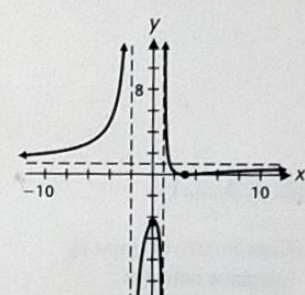
Sixth-degree polynomial  
 $H(x) = x^6 - 7x^4 + 12x^2 - x - 2$



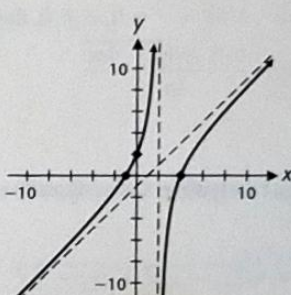
$$f(x) = \frac{1}{x}$$



$$f(x) = \frac{2x}{x-3}$$

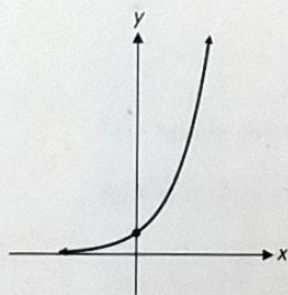


$$f(x) = \frac{x^2 - 6x + 9}{x^2 + x - 2}$$



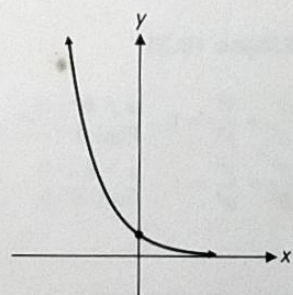
$$f(x) = \frac{x^2 - 3x - 4}{x - 2}$$

### CHAPTER 4 EXPONENTIAL AND LOGARITHMIC FUNCTIONS



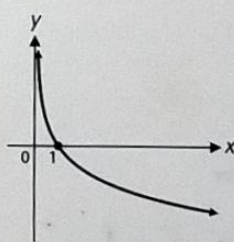
$$y = b^x$$

$$b > 1$$



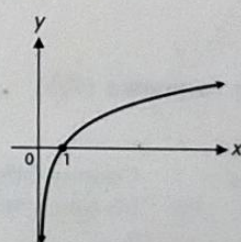
$$y = b^x$$

$$0 < b < 1$$



$$y = \log_b x$$

$$0 < b < 1$$



$$y = \log_b x$$

$$b > 1$$



## Inequalities and Intervals (A.1)

$a < b$	$a$ is less than $b$
$a \leq b$	$a$ is less than or equal to $b$
$a > b$	$a$ is greater than $b$
$a \geq b$	$a$ is greater than or equal to $b$
$(a, b)$	Open interval; $\{x \mid a < x < b\}$
$(a, b]$	Half-open interval; $\{x \mid a < x \leq b\}$
$[a, b)$	Half-open interval; $\{x \mid a \leq x < b\}$
$[a, b]$	Closed interval; $\{x \mid a \leq x \leq b\}$

## Quadratic Formula (2.5)

If  $ax^2 + bx + c = 0$ ,  $a \neq 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Rectangular Coordinates (A.2, A.3, 2.1)

$(x_1, y_1)$	Coordinates of point $P_1$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint of line joining $P_1$ and $P_2$
$m = \frac{y_2 - y_1}{x_2 - x_1}$ , $x_1 \neq x_2$	Slope of line through $P_1$ and $P_2$

## Arithmetic Sequence (7.3)

$a_1, a_2, \dots, a_n, \dots$	
$a_n - a_{n-1} = d$	Common difference
$a_n = a_1 + (n-1)d$	$n$ th-term formula
$S_n = a_1 + \dots + a_n = \frac{n}{2}[2a_1 + (n-1)d]$	Sum of $n$ terms

$$S_n = \frac{n}{2}(a_1 + a_n)$$

## Geometric Sequence (7.3)

$a_1, a_2, \dots, a_n, \dots$	
$\frac{a_n}{a_{n-1}} = r$	Common ratio
$a_n = a_1 r^{n-1}$	$n$ th-term formula
$S_n = a_1 + \dots + a_n = \frac{a_1 - a_1 r^n}{1 - r}$ , $r \neq 1$	Sum of $n$ terms

$$S_n = \frac{a_1 - r a_n}{1 - r}, \quad r \neq 1$$

$$S_\infty = a_1 + a_2 + \dots = \frac{a_1}{1 - r}, \quad |r| < 1 \quad \text{Sum of infinitely many terms}$$

## Permutations and Combinations (7.4, 7.6)

$n! = n(n-1) \cdots 2 \cdot 1$ ,  $n \in \mathbb{N}$   $n$  factorial

$$0! = 1$$

For  $0 \leq r \leq n$ ,

$$P_{n,r} = \frac{n!}{(n-r)!} \quad \text{Permutation}$$

$$C_{n,r} = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \text{Combination}$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad n \geq 1 \quad \text{Binomial formula}$$

## Circle (A.3)

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Center at } (h, k); \text{ radius } r$$

$$x^2 + y^2 = r^2 \quad \text{Center at } (0, 0); \text{ radius } r$$

## Parabola (2.3, 8.1)

$$y^2 = 4ax, \quad a > 0, \text{ opens right; } a < 0, \text{ opens left}$$

Focus:  $(a, 0)$ ; Directrix:  $x = -a$ ;

Axis:  $x$  axis

$$x^2 = 4ay, \quad a > 0, \text{ opens up; } a < 0, \text{ opens down}$$

Focus:  $(0, a)$ ; Directrix:  $y = -a$ ;

Axis:  $y$  axis

## Ellipse (8.2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0 \quad \text{Foci: } F'(-c, 0), F(c, 0); c^2 = a^2 - b^2$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad a > b > 0 \quad \text{Foci: } F'(0, -c), F(0, c); c^2 = a^2 - b^2$$

## Hyperbola (8.3)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{Foci: } F'(-c, 0), F(c, 0); c^2 = a^2 + b^2$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{Foci: } F'(0, -c), F(0, c); c^2 = a^2 + b^2$$

## Translation Formulas (8.4)

$$x = x' + h, y = y' + k; \quad x' = x - h, y' = y - k$$

New origin  $(h, k)$



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